J. Membrane Biol. 32, 197–199 (1977)

Letter to the Editors

Diffusion and $1/f$ Noise. Part II

Received 3 January 1977

Diffusion has been examined recently as a source of $1/f$ noise by Frehland (1976), Mikulinsky and Mikulinsky-Fishman (1976), Neumcke (1975), and myself (Green, 1976). In the note preceding this, Frehland (1977) raises a point concerning my paper which requires some clarification. In essence, he notes that fluctuations in *flux* need not be proportional to fluctuations in *concentration*. A number of authors (e.g., van Vliet and Fassett, (1965)) have calculated fluctuations in concentration, and used this to interpret data; it is to this that Frehland objects. However, we should note that the measurements on artificial membranes which are relevant here (Dorset and Fishman (1975) and Green (1976) are actually measurements of *voltage* fluctuations, made at constant current. As long as the concentration fluctuations remain small, the voltage and flux fluctuations also remain small. The following argument (*see*, for example, Feher & Weissman, 1973) then holds for voltage fluctuations at constant current in a one dimensional capillary, or pore:

$$(\delta V)^2 = I_{DC}^2 (\delta R)^2 = V_{DC}^2 \frac{(\delta R)^2}{R^2}$$

$$= V_{DC}^2 \frac{(\delta N)^2}{N^2} \left(\frac{N}{R} \frac{\partial R}{\partial N}\right)^2. \tag{1}$$

Here, $(\delta V)^2$ = mean square fluctuation, I_{DC}, V_{DC} are the mean current and voltage, R the mean resistance, $(\delta R)^2$ the mean square resistance fluctuation, N the mean concentration, $(\delta N)^2$ the mean square concentration fluctuation. The final term in the product, $\left(\frac{N}{R} \frac{\partial R}{\partial N}\right)^2$, is of order unity, and would be unity if conductivity were exactly proportional to concentration. The result is that $(\delta V)^2$ is proportional to $(\delta N)^2$ for the case involved here. Hence, for the slopes of the power spectra, the objection raised by Frehland turns out not to be important.

As far as Frehland's calculation itself (Frehland, 1976), the infinite case does not apply to the experiments involved. In the semi-infinite case, Frehland finds the spectral density of concentration and flux to be proportional to each other. In general, he, van Vliet and Fassett (1965), and the other authors cited, find f^{-0} or $f^{-1/2}$ at low frequency, and $f^{-3/2}$ at high frequency.

The observation of $1/f$ noise in membranes with a distribution of pore lengths suggests averaging over such a distribution. This has the effect of combining the f^{-0} or $f^{-1/2}$ segments of the spectra of some pores with the $f^{-3/2}$ segments due to other pores, not surprisingly providing an approximately f^{-1} spectrum. To insure that this is reasonable, it is necessary to check membranes without a distribution, to observe the $f^{-3/2}$ behavior, as was done (Green, 1976). Even though the f^{-1} behavior results from diffusion, as Frehland and 1 agree, it is not necessary to calculate f^{-1} directly from the transport equations.

It is worth noting that the Mikulinsky and Mikulinsky-Fishman (1976) paper carries the calculation further than Frehland, or the van Vliet and Fassett paper, which 1 quoted; in particular, they explicitly related concentration to potential via the Poisson equation. Their calculation is still linear, and the final result not significantly different from the others. (Neumcke (1975), by including the effect of gates, appears to be only author to have produced a model which may relate diffusion noise to biological $1/f$ noise.)

Finally, there is the question of actual nonlinearity. (Frehland uses "linearly coupled" to mean proportional, not linear. The Nernst-Planck flux equation will be linear unless field E, or diffusion constant D, are functions of concentration, which they are not, in Frehland's calculation). Several nonlinear calculations leading to $1/f$ noise over a finite range (e.g., Tchen, 1973*a, b*) have appeared. These involve, in one way or another, cascading of the energy of turbulence across the spectrum. The question of nonlinearity is an important one, very possibly relevant biologically, but is not addressed in any of the papers discussed above.

References

Dorset, D.L., Fishman, H.M. 1975. Excess electrical noise during current flow through porous membranes separating noise solutions. *J. Membrane Biol.* **21**:291

Feher, G., Weissman, M. 1973. Fluctuation spectroscopy: Determination of chemical reaction kinetics from the frequency spectrum of fluctuations. *Proc. Nat. Acad. Sci. USA* **70**:870

Frehland, E. 1976. $1/f$ noise generated by constrained diffusion through a flux controlling boundary. *Z. Naturforsch.* **31a**:942

Frehland, E. 1977. Diffusion as a source of $1/f$ noise. *J. Membrane Biol.* **32**:195

Green, M.E. 1976. Diffusion and $1/f$ noise. *J. Membrane Biol.* **28**:281

Mikulinsky, M.A., Mikulinsky-Fishman, S.N. 1976. The theory of $1/f$ noise. *Phys. Lett.* **58A**:46

Neumcke, B. 1975. $1/f$ membrane noise generated by diffusion processes in unstirred solution layers. *Biophys. Struct. Mech.* **1**:295

Tchen, C.M. 1973*a*. Repeated cascade theory of homogeneous turbulence. *Phys. Fluids* **16**:13

Tchen, C.M. 1973*b*. Repeated cascade theory of turbulence in an homogeneous plasma. *Phys. Rev.* **A8**:500

Vliet, K.M. van, Fassett, J.R. 1965. Fluctuations due to electronic transitions and transport in solids. *In:* Fluctuation Phenomena in Solids. R.E. Burgess, editor Ch. VII. Academic Press, New York

Michael E. Green
Department of Chemistry
The City College of The City University of New York
New York, NY 10031